LAMBERTIAN SURFACES, TROLANDS

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THIS CHAPTER WILL INCLUDE A REVIEW OF:

- Luminance and illuminance
- Lambertian surfaces
- Retinal illumination
- Blackbody radiators and color temperature

LUMINANCE AND ILLUMINANCE

The photometric units that we will use most frequently are those for luminance and illuminance. Both applyto extended sources or surfaces. The basic metric unit for luminance is the nit.

1 nit = 1 candela/m^2

Luminance describes how much luminous intensity is given off by an extended source or surface (in terms of projected surface area), which correlates with how bright it looks. An interesting property of luminance is that it does not change with viewing distance.

Q. Can you explain why the luminance of an object doesn't change with distance?
A. _____

Illuminance describes the amount of light falling onto a surface. The basic metric unit is the lux. The illuminance decreases with the inverse of the square of the distance *(inverse square law).* For example, if you increase the distance from a light source by a factor of two, the illuminance decreases by a factor of 1/4. Furthermore, a surface will receive maximum illumination if the surface is perpendicular to the light source. If the surface is tilted, the illuminance will decrease in proportion to the cosine of the surface tilt, relative to the perpendicular.

1 lux = 1 lumen/m^2

LAMBERTIAN SURFACES

Most objects we see are not self luminous, but are illuminated by some other light source – we see the light reflected off the object. Highly polished surfaces cause **specular** reflections. In the case of specular reflections, most of the light will reflect off the surface in only one direction. Examples include mirrors or metallic objects that look shiny.

Other objects do not appear shiny; they have a matte appearance because they reflect light in all directions. These surfaces are known as **diffuse** reflectors, **Lambertian surfaces** or **cosine diffusers**. **Lambert's cosine law** states that the luminous intensity in any direction is directly proportional to the cosine between that direction and the direction of incidence (angle θ in Figure 6-1).

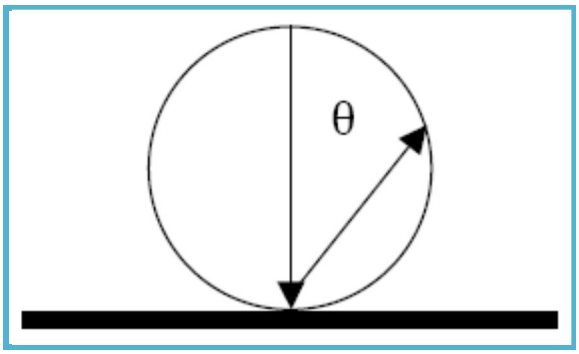

Figure 6-1: *Lambert's cosine law for diffuse reflectors*

When looking at a diffusely reflecting surface such as a piece of paper, its luminous intensity decreases with the cosine of the viewing angle, but the projected area (size) of the paper also decreases with the cosine of the viewing angle. This leads to an interesting property of Lambertian surfaces: *When viewed from any angle, the luminance (brightness) of a Lambertian surface remains constant.*

RELATIONSHIP BETWEEN ILLUMINANCE AND LUMINANCE

For a Lambertian surface that reflects 100% of the incident light (i.e., a piece of pure white paper), the relationship between illuminance in lux and luminance in nits is illustrated in Figure 6-2.

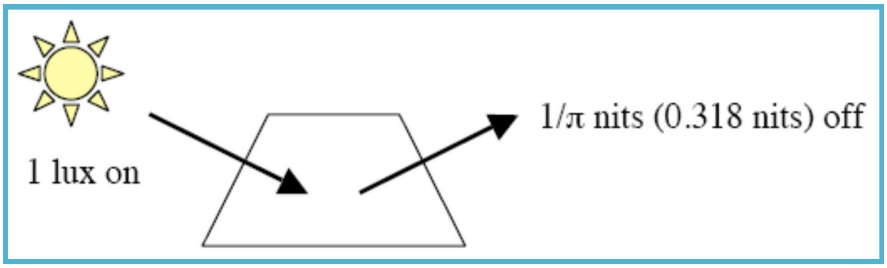

Figure 6-2: *For a 100% reflecting Lambertian surface, 1 lux of illuminance produces 1/π nits of luminance.*

To simplify luminance computations with Lambertian surfaces, another unit of luminance, the apostilb was created. One apostilb is equal to 1/π nits, so when luminance is measured in apostilbs, the relationship between illuminance and luminance is as shown in Figure 6-3.

1 apostilb = 1/π candelas/m^2 = 1/π nits

LAMBERTIAN SURFACES (CONT.)

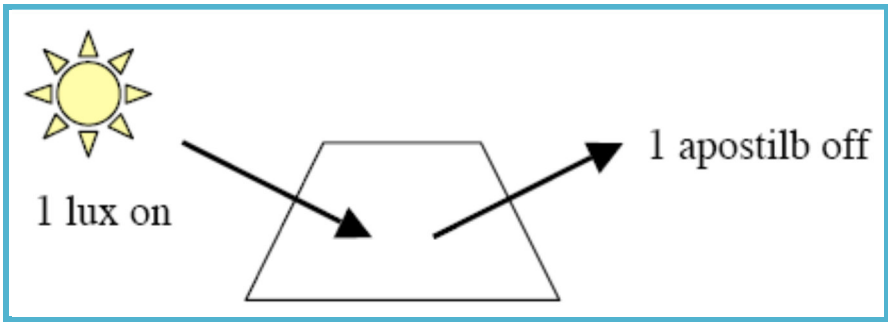

Figure 6-3: *The relationship between illuminance and luminance when apostilbs are used for luminance. The 1:1 relationship between lux and apostilbs applies only for a 100% reflecting Lambertian surface. For a 50% reflecting surface, the relationship is direct, but the lux : apostilb ratio would become 2:1, as illustrated in Figure 6-4*

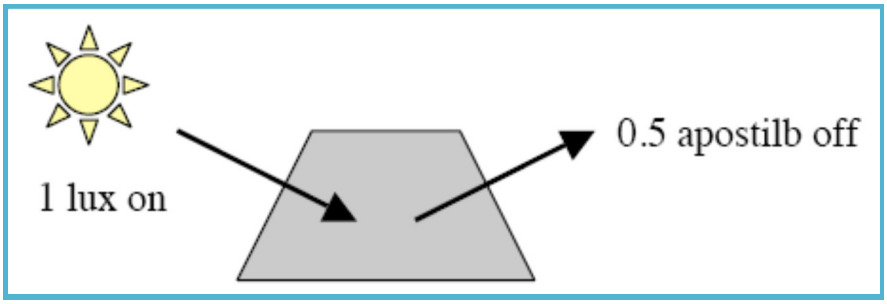

Figure 6-4: *Illustration of a Lambertian surface that reflect 50% of the incident light*

The English units for illuminance and luminance (for Lambertian surfaces) are summarized in Table 6.1, below. Notice how they parallel the metric units. When English units are used, the relationship between footcandles of illuminance and footlamberts of luminance is the same as that shown in Figure 6.3, except that the units are footcandles (illuminance) and footlamberts (luminance); that is, 1 footcandle on produces 1 footlambert off.

Table 6-1: *English and metric units for illuminance and luminance.*

PHOTOMETRIC QUANTITY	ENGLISH UNIT	METRIC UNIT
Illuminance	Footcandle 1 lumen/ft^2	Lux 1 lumen/m^2
Luminance (Lambertian surfaces)	Footlambert 1/π candelas/ft^2	Apostilb 1/π candelas/m^2

Note: *That these examples assume that the Lambertian surface reflects 100% of the light. Not all Lambertian surfaces reflect 100% of the light. For example, a gray Lambertian surface might reflect 50% of the light, and the resulting luminance would be reduced by 50%*

LAMBERTIAN SURFACES (CONT.)

RETINAL ILLUMINANCE

Retinal illuminance refers to the amount of light falling on the retina, such as for the retinal image of some object. It remains constant for any object distance - if you look at an object, it does not become dimmer as it gets further away. However, according to the inverse square law, the illuminace reaching an eye decrease with the square of the distance. The area of the retinal image also decreases with the square of the distance, thus retinal illuminance remains constant.

Retinal illuminance is directly proportional to object luminance and pupil area, and is measured by a simple formula:

Troland = (luminance of the object in nits) x (pupil area in mm^2)

A common mistake in retinal illuminance calculations is to use pupil diameter rather than pupil radius when computing the area. The following examples show how to compute retinal illuminance.

Example 1

Suppose you are looking at a piece of paper in bright sunlight. What would the retinal illuminance be for the image of that paper on your retina, assuming the luminance of a piece of paper in sunlight is about 10,000 nits (refer to Schwartz, 2004 Table 3-1) and your pupil diameter is equal to 2.0 mm?

Pupil area = $\pi r^2 = (\pi 1^2) = 3.14\ mm^2$

Retinal illuminance = (10,000 nits) $\times$ (3.14 mm^2) = 31,400 trolands

Example 2

What would the retinal illuminance be if you are looking at a piece of white paper in starlight (luminance = 0.0001 nits; refer to Schwartz, 2004 Table 3-1) and the pupil diameter is equal to 8 mm?

Pupil area = $\pi r^2 = (\pi 4^2) = 50\ mm^2$

Retinal illuminance = (0.0001 nits) $\times$ (50 mm^2) = 0.005 trolands

BLACKBODY RADIATORS AND COLOR TEMPERATURE

Incandescent light bulbs produce light when electricity passes through the filament and heats it, causing it to glow. You might assume that a white light contains an equal amount of radiant power at all wavelengths, but this is not necessarily true. Some 'white' lights produce relatively more power in the long wavelengths, while other lights produce relatively more short-wavelength content. The relative energy produced by a light source across the visible spectrum can be plotted, which shows three standard white light sources. Illustrated are standard illuminants A, B and C. Illuminant A represents the typical household incandescent light bulb. It produces more energy at the long wavelengths and therefore has a subtle reddish tint. **Standard illuminant C** is recommended for many color vision tests since it is relatively flat across the middle of the visible light range.

The energy spectrum produced by a light source can be specified by the plots mentioned above, or by its **color temperature**. Each standard illuminant has its own color temperature. Associated with each color temperature is a specific energy spectrum, and this is based on the concept of a **blackbody radiator**.

A blackbody radiator is a hypothetical object that glows when heated and emits electromagnetic radiation. The exact spectrum of the energy produced depends on the temperature of the blackbody radiator. For lower temperatures, it will glow with predominantly longer wavelengths; as you increase the temperature, the peak wavelength (wavelength with highest energy) gradually shifts to shorter wavelengths. Imagine heating a piece of steel. At first it glows red, but with increasing temperature it turns orange, then yellow, and then blue.

BLACKBODY RADIATORS AND COLOR TEMPERATURE

This is expressed in **Wein's displacement law**, which states that the peak wavelength is inversely proportional to the temperature. The **Stefan-Boltzman law** says that the total power contained in the source is proportional to the temperature raised to the fourth power. The peak of the energy spectrum for a blackbody radiator with different temperatures shifts with increasing temperature, and the area under the curves (total power) also increases with temperature. Some incandescent light sources produce an energy spectrum similar to a blackbody radiator with a certain temperature, and this is referred to as the color temperature of the light source. This is a convenient way to specify the spectral output of a light source.

Our visual system does a good job of compensating for differences in color from different white light sources, but cameras sometimes have a harder time. In order to get the correct color balance for "white" light sources you may have to adjust the 'white balance' setting. Figure 6.5 shows 4 pictures of the same scene taken with different white balance settings. The room was illuminated by fluorescent lights, but also received much light from the windows. It was a cloudy, overcast day at about 11:00 AM in Eastern Oklahoma. Try to explain why they various settings cause the particular colored tint for the photo.

FIGURE 6-5

Figure 6-5 (a): *Incandescent*

Figure 6-5 (b): *Fluorescent*

Figure 6-5 (c): *Cloudy*

Figure 6-5 (d): *Daylight*

FILTERS (COLORED AND NEUTRAL DENSITY)

Schwartz, 2004 Chapter 4, pages 79-85 reviews colored and neutral density filters.
Study those sections on your own.

SELECTED READING/REFERENCES

- Schwartz SH. **Visual Perception - A Clinical Orientation, 3rd Edition**. Appleton & Lange, Stamford, Connecticut, 2004