THIN LENSES I

AUTHORS

Michael Smith: Cape Peninsula University of Technology

PEER REVIEWERS

Dr B. Ralph Chou: Professor Emeritus, School of Optometry and Vision Science, University of Waterloo

Dr Vincent Nourrit: The University of Manchester

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TERMINOLOGY OF THIN LENSES

A thin lens is one whose thickness is very small in comparison to its focal length[1]. A typical thin lens consists of a piece of glass or plastic, ground so that each of its two refracting surfaces is a portion of either a sphere or a plane.

A lens is used to direct or control rays of light. The refraction of light at the surface of a lens depends on its shape, its index of refraction, and the nature of the medium surrounding it (usually air), in accordance with Snell's Law. Lenses are commonly used to form images by refraction in spectacles and in optical instruments such as cameras, telescopes and microscopes.

[1] Although any contact lens must be considered a thick lens

Lenses can be placed in two groups. Those in the one group are thicker at the centre than at the rim. They are known as **converging lenses** (or **convex lenses**). They refract incident parallel rays so that they converge at a point on the opposite side of the lens.

The others are thinner at the centre than at the rim. They are known as **diverging** or **concave lenses.** They refract incident parallel rays so that they appear to diverge from a point located on the incident side of the lens. Any object viewed through such a lens always appears erect and smaller than when viewed with the unaided eye. A **meniscus** (meaning crescent-shaped) lens has one convex surface and one concave surface.

The *principal axis* or **optical axis** of a lens is the line joining the centres of curvature of the two surfaces. Rays making small angles with the optical axis are referred to as **paraxial** rays. The **optical centre** of a thin lens is such that **any ray passing through that point is not deviated**. For the cases we will deal with in this course, the optical centre of a lens can conveniently be thought of as the geometric centre of the lens.

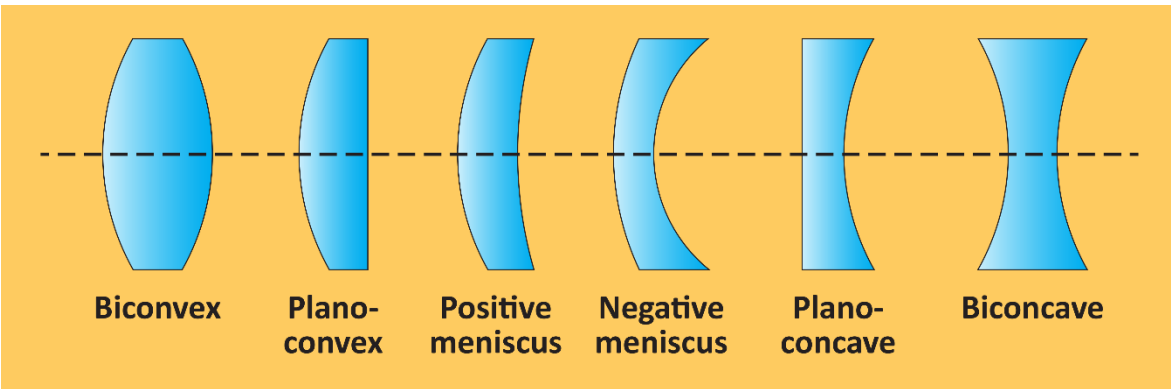

Figure 4.1: *Different types and shapes of lenses*

FOCAL LENGTHS

There are two points on the optical axis which need special attention. The following figure shows a pencil of paraxial rays from an object at an infinite distance from a converging lens. As such, the pencil will be effectively parallel on reaching the lens. These rays are refracted through the point F_2 which is called the *second principal focus* or *second focal point* of the lens. The distance from the optical centre to F_2 is called the *second focal length* of the lens and will be referred to by the symbol f_2.

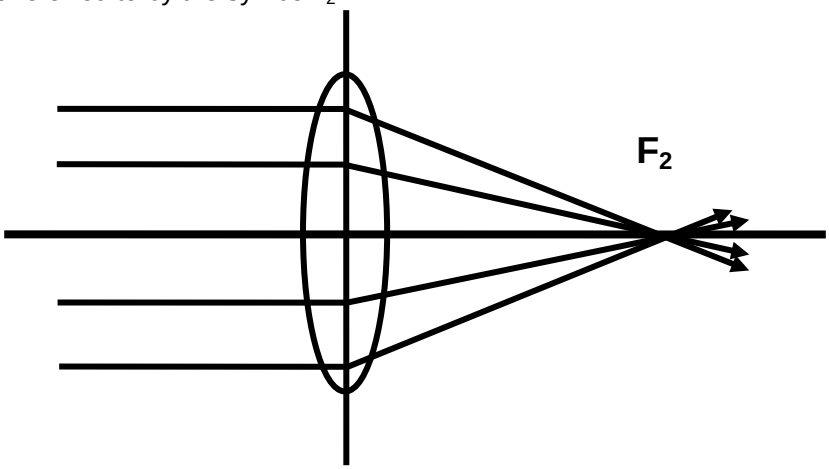

Figure 4.2: *Illustration of the second focal point of a converging lens*

Similarly, in the next figure, there is the point F_1 from which paraxial rays, after refraction by the lens, emerge parallel to the axis. F_1 is referred to as the *first principal focus* or *first focal point* of the lens, and the distance from the optical centre to F_1 is referred to as the *first focal length* of the lens and is given the symbol f_1.

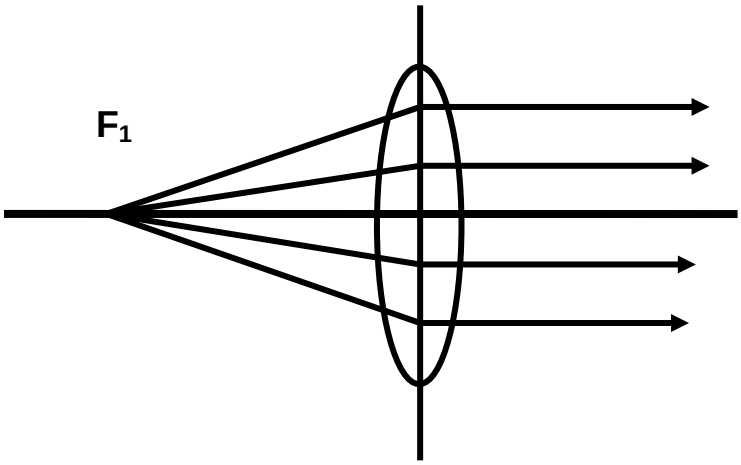

Figure 4.3: *Illustration of the first focal point of a converging lens*

If the lens is thin, and the medium on each side of the lens is the same, then the numerical values of f_1 and f_2 will be identical.

Note that the diagrams show lenses with exaggerated thicknesses and rays that make greater angles with the optical axis than should be the case for paraxial rays. This has been done simply to allow the clear exposition of the refraction at each surface.

When "**the focal length**" of the lens is stated without specifying it to be the first or second focal length, it must always be inferred that this means the **second focal length** of the lens.

In the case of a diverging lens, there are two similar principal foci. The next figure shows the point F_1, the *first principal focus* or *first focal point* of a diverging lens. Rays directed towards this point emerge from the lens parallel to the optical axis. Once again, the distance from the optical centre to F_1 is the first focal length, f_1. F_1 can be seen to be on the opposite side of the lens compared to the incident light.

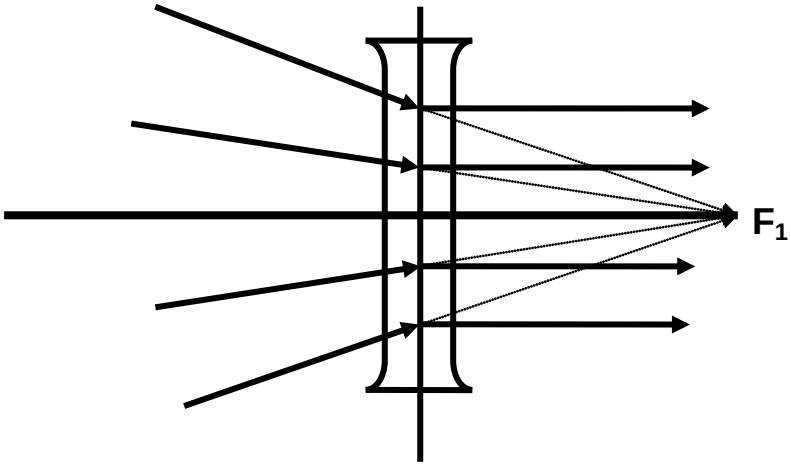

Figure 4.4: *Illustration of the first focal point of a diverging lens*

Similarly, the *second principal focus* or *second focal point* (designated F_2) is formed where rays that are incident on a diverging lens parallel to the optical axis appear to diverge from. The distance from the optical centre to F_2 is thus the *second focal length* of the lens. The figure shows that the second focal point lies on the same side as the incident light.

As before, if the lens is thin and the media on each side of the lens are identical in refractive index, then the focal lengths have the same magnitude.

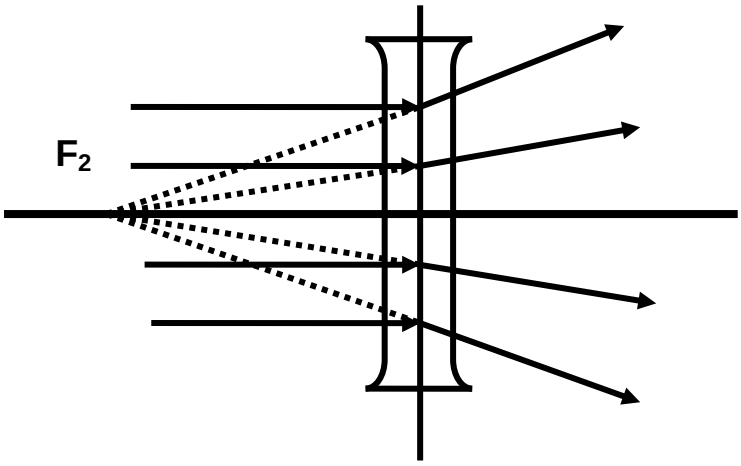

Figure 4.5: *Illustration of the second focal point of a diverging lens*

REAL AND VIRTUAL

The major usefulness of lenses is their ability to form images of an object. The object may be self-luminous, giving off its own light (like the sun or a light bulb), or it may reflect the light that falls on it (like an apple or a page of this book). In either case, an image of the object is formed where light rays that come from points on the object intersect or at the points from which the rays appear to originate.

When the light rays actually intersect at the image, we call this a **real image.** A screen placed at a real image point would show the image in the same way that pictures appear on a movie screen.
A **virtual image** is formed at a point where the light appears to converge, or from which it appears to come. **If you place a screen at the position of a virtual image, no image is observed on the screen**, for the light rays do not actually intersect there. An example of images in a mirror may help to explain virtual images. If you stand one metre in front of a mirror, your image appears to lie one metre behind the mirror. However, you will not find an image on a screen placed at the image position one metre behind the mirror.

From the diagrams in the preceding section, it can be seen that a **converging** lens has a **real** second focus. In contrast, a diverging lens has a virtual second focus.

If the image and the object are on opposite sides of the optical axis, we describe the image as being **inverted** with respect to the object. Conversely, if the image and the object are on the same side of the optical axis, we describe the image as being **erect** with respect to the object.

IMAGE FORMATION BY GRAPHICAL CONSTRUCTION

An object reflects light rays in all directions from all points on its surface. We illustrate this in the following figure by using arrows from the tip of the pencil to represent light rays. However, to locate the image of the pencil formed by a thin lens, you don't need to trace the light rays from all over the object. Instead, you can locate the image by tracing three particular rays from a point on the object. In each case the focal length of the lens and the distance from the object to the centre of the lens (the **object distance**) determine the distance of the image from the lens (the **image distance**).

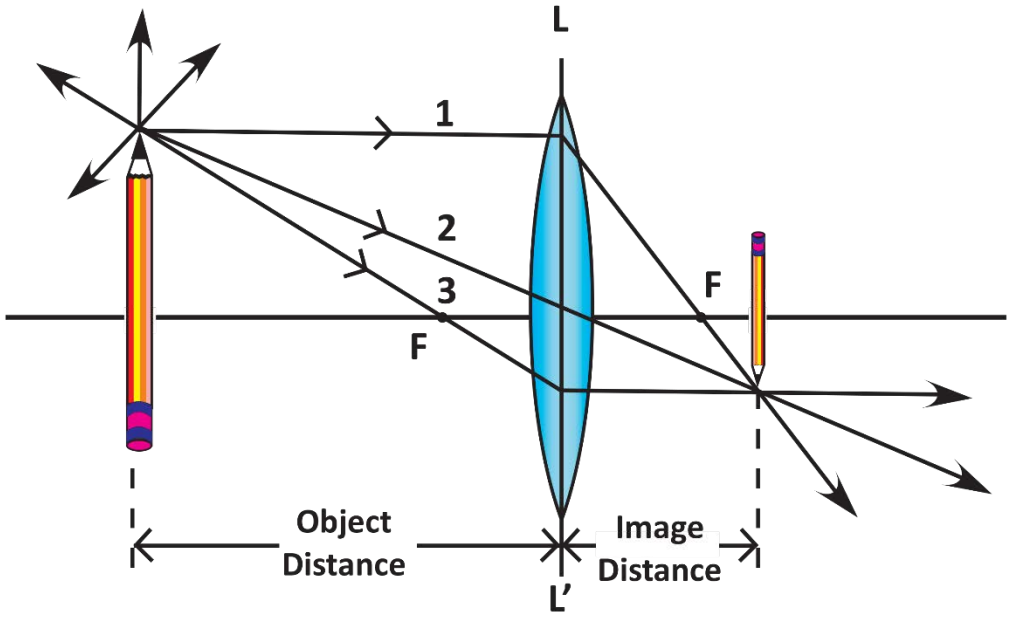

Figure 4.6: *Image formation by graphical construction*

CONVERGING LENS

The three rays that can conveniently be used arise from the definitions considered at the beginning of this chapter. Refer to the following diagram to see how these definitions are used. By convention, optical drawings assume that the light travels from the left to the right in a positive direction. This sign convention is expanded in the Thin Lenses 2 chapter.

1. **Ray 1** is incident on the lens parallel to the optical axis and is therefore refracted through the second focal point.
2. **Ray 2** passes through the optical centre, undeviated.
3. **Ray 3** passes through the first focal point and therefore leaves the lens parallel to the optical axis.

Although we can use these three specific rays to locate the position of the image, the actual formation of the image is much more complex. The light from each point on the object converges to form the corresponding point on the image. Rays from a single point on the object strike the entire lens surface. Conversely, any small area of the lens surface is struck by rays coming from every point on the object. All of the rays striking each small area are focused to form the image. **Thus the image is composed of contributions of light that pass through every part of the lens**. For this reason, blocking out part of the lens does not cast a shadow on the image, but simply reduces the brightness of the image by restricting the amount of light that gets to it.

When the rays are close to and nearly parallel to the axis (the paraxial approximation), a small planar object perpendicular to the optical axis is imaged by a thin lens into a planar region that is also perpendicular to that axis. Thus all points in the object plane are represented as points in an image plane. When the object is three-dimensional, like the pencil in the above figure, points located in different object planes do not focus in a single image plane. We will restrict our discussion to planar objects and their planar images. Thus we usually represent objects and images with simple arrows perpendicular to the optical axis.

Let us state the **graphical procedure** to follow for locating the image from a given lens when you know the object's position. We can also determine the object size from such a diagram.

A. Select an appropriate scale and mark the position of the lens on the optical axis. Bear in mind that the image formed by a diverging lens will be formed between the object and the lens, (on the same side as the incident light).

B. Draw a line LL′ to represent the lens surfaces.

C. Mark the first and second focal points of the lens on the optical axis and locate the object on the axis, all to the same scale.

D. Draw rays 1, 2 and 3. Any two of these rays are sufficient to locate the image, but you should always draw the third ray as a check, if possible.

E. If the rays converge on the side opposite the incident light, the image is real and is located at the intersection of these three rays. If the rays diverge, the image is virtual and is located where these three rays appear to originate. You can measure the position and image height directly from the scale drawing.

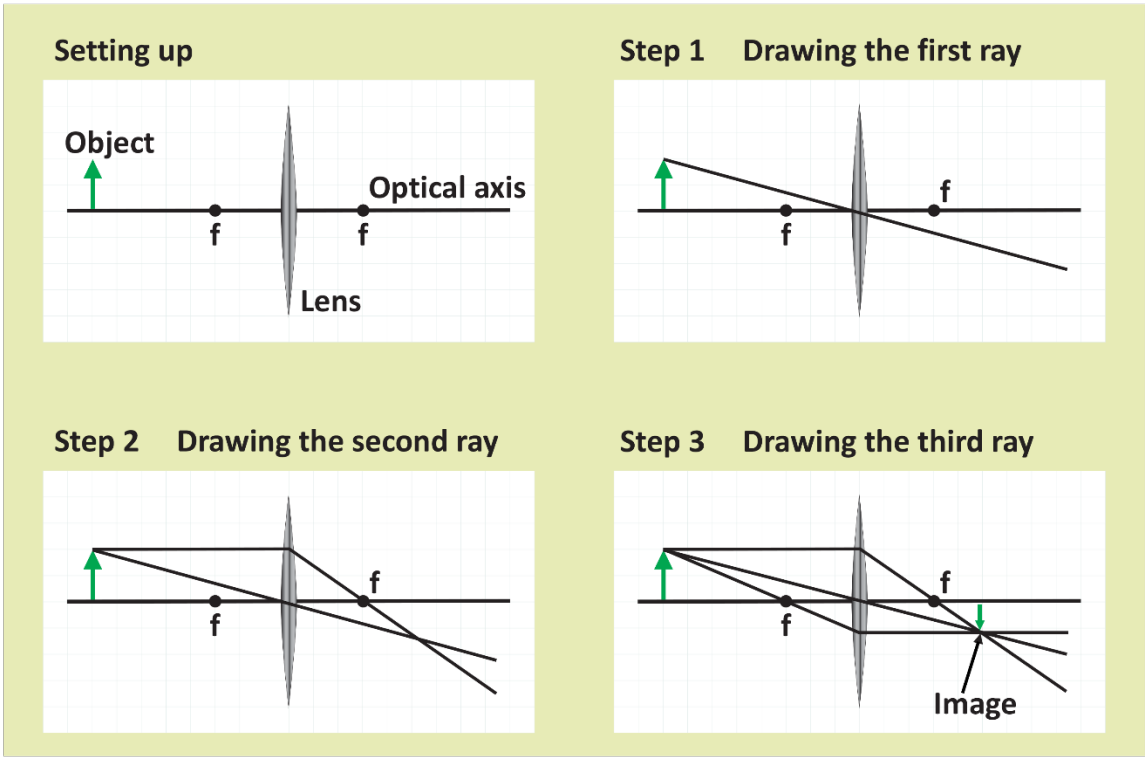

Figure 4.7: *Image formation by graphical construction, principal rays*

You could make a ray diagram for each point on the object to fully locate the image. However, if the whole of the object is at approximately the same distance from the lens, then you need only consider the image of one point of the object and the rest of the object is thereby located.

Again, note that for a given object distance, the size and position of the image are completely determined by the focal length of the lens. Therefore, if the object size is not given, you may use any convenient size.

The rays used in the case of a diverging lens are similar to the ones considered previously.

DIVERGING LENS

1. **Ray 1** is incident on the lens parallel to the optical axis and therefore appears to arise from the second focal point.
1. **Ray 2** passes through the optical centre, undeviated.
1. **Ray 3** is directed towards the first focal point and therefore leaves the lens parallel to the optical axis.

Note that in this case, you need to extend the rays to find the point where they appear to intersect (virtual image).

LINEAR MAGNIFICATION

The **linear magnification**, m, of an optical system is the ratio between the size of the image and that of the object. A magnification of exactly 1 means that the image and the object are the same size. Similarly, if the image is larger than the object the magnitude of the magnification is greater than 1. By "size" we mean any **linear** dimension such as height or width.

$$m = \frac{\text{Image Size (I)}}{\text{Object Size (O)}}$$

If we use the sign convention specifying that distances above the optical axis are considered positive and that distances below are considered negative, we obtain the following result:

If the image is **erect** with respect to the object, the magnification is **positive**.

If the image is **inverted** with respect to the object, the magnification is **negative.**

CONJUGATE POINTS

Consider the figure 4.8 below. An object at point **O** is imaged at point **I**. According to the Principle of Reversibility, any actual ray of light in an optical system, if reversed in direction, will retrace the same path backward. Therefore, an object placed at position **I** will give rise to an image at position **O**. The points O and I are therefore interchangeable (light emitted at one of the points will be focused at the other) and are referred to as *conjugate points.*

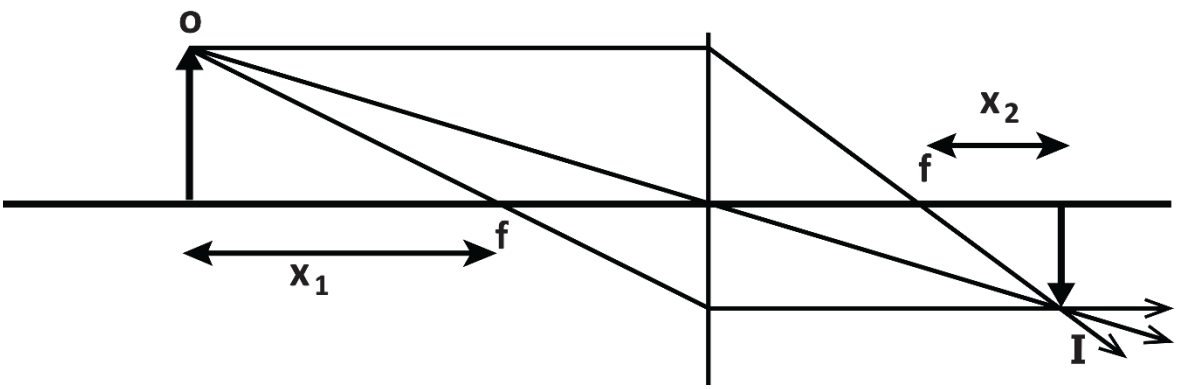

Figure 4.8: *Conjugate points*