INTERFERENCE

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INTERFERENCE

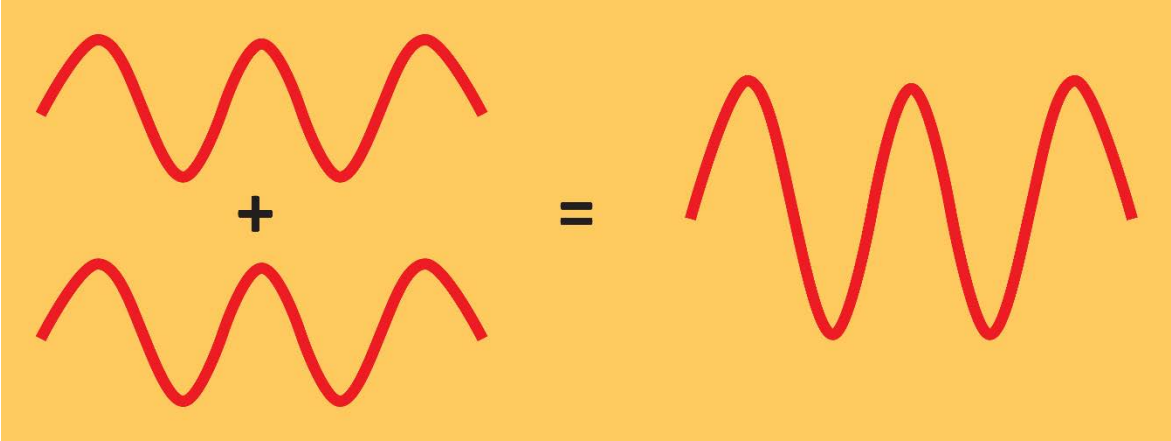
When two or more waves arrive at a point in a medium simultaneously, the *principle of superposition* can be used to find the resultant displacement.

The superposition principle states that when two or more waves traverse the same space, the net amplitude at each point is the sum of the amplitudes of the individual waves. This interaction of waves that exist at a point at the same time is known as *interference*. The same holds for electromagnetic waves moving through a medium.

The amplitudes can add together either constructively or destructively. In constructive interference, the amplitude of the resultant wave is greater than that of either of the individual waves, while in destructive interference, the resultant amplitude is less than that of either of the individual waves. In the case of two waves of ***equal amplitude***, **complete constructive and complete destructive interference may occur and will be illustrated first (figure 7.1).**

Fundamentally, all interference associated with light waves arises as a result of combining the fields that constitute the individual waves.

COMPLETE CONSTRUCTIVE INTERFERENCE.



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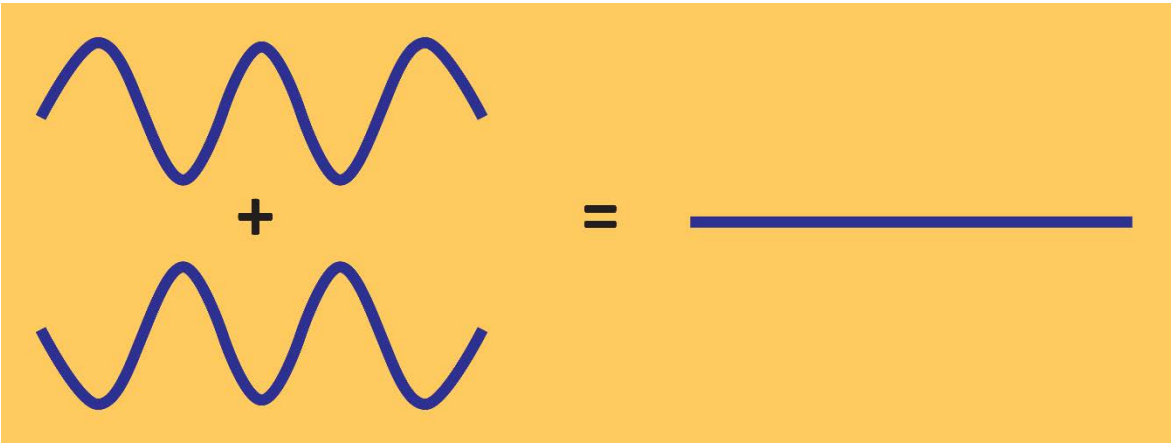

INTERFERENCE DUE TO WAVES PARTIALLY IN PHASE.

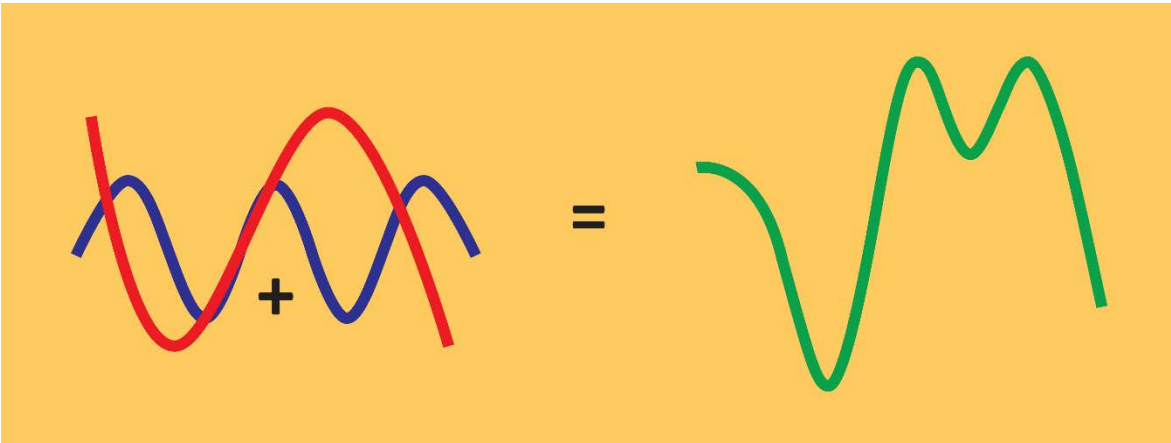

Figure 7.1: *Constructive and destructive interference*

Interference effects in light waves are not easy to observe because of the short wavelengths involved. These short wavelengths mean that they have **very high frequencies and therefore there are many waves interacting per second**.

In order to observe sustained interference in light waves from more than one source, the following conditions must be met:

1. The sources must be **coherent**, that is, they must maintain a constant phase with respect to each other.

2. The sources must emit identical wavelengths. (This condition is easiest to meet if the sources are monochromatic.)

If two separate light sources are placed side by side, no interference effects are **observed** because in this case the light waves emitted by each of the sources are emitted independently of the other source. Hence, their emissions do not maintain a constant phase relationship with each other over the time of observation. Light from an ordinary light source undergoes such random changes about once every 10^{-8} seconds. Therefore the conditions for constructive interference, destructive interference, or some intermediate state last for times of the order of 10^{-8} seconds. The result is that no interference effects are observed since the eye cannot follow such short-term changes. Such light sources that do not maintain a constant phase relationship are said to be *incoherent*.

Please note that interference phenomenon can be observed with incoherent "white" (i.e. polychromatic) light (e.g. colour patterns on soap bubbles, anti reflections coatings on spectacles, etc.) but the conditions for them to be observed are much more strict (and difficult to put in place) than with coherent light.

YOUNG'S DOUBLE-SLIT EXPERIMENT

The phenomenon of interference in light waves from two sources was first demonstrated by Thomas Young in 1801.

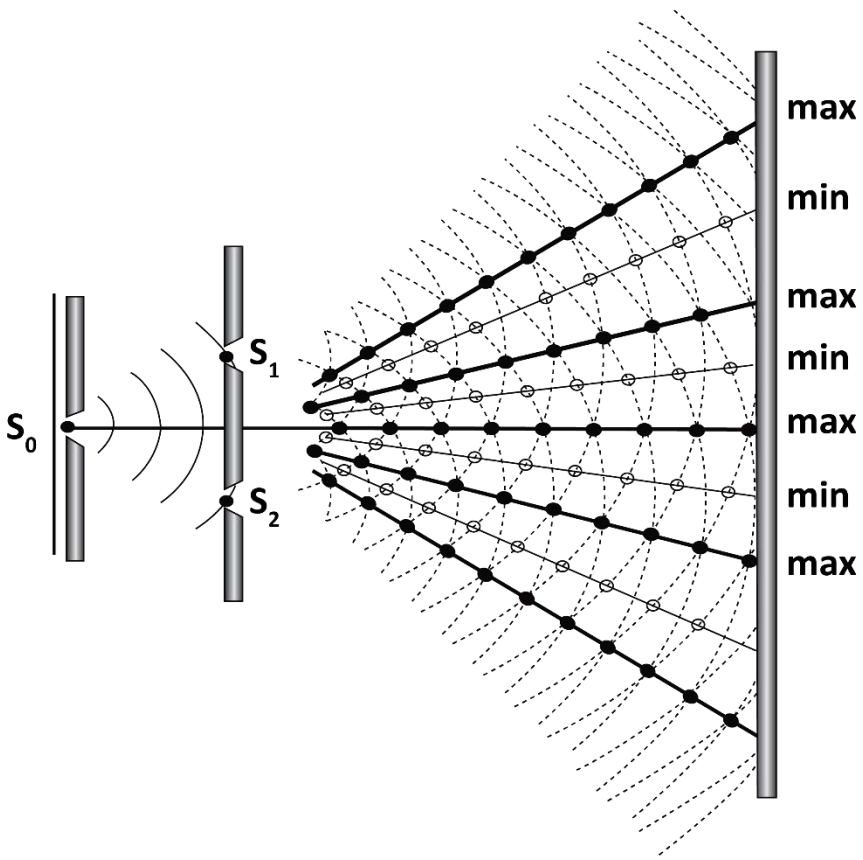

Figure 7.2: *Young's double slit experiment*

Sunlight is incident on a screen, which is provided with a narrow slit S_0. The waves emerging from this slit arrive at a second screen, which contains two narrow, parallel slits, S_1 and S_2. Since these slits are equidistant from the first slit, they serve as a pair of coherent light sources because waves emerging from them originate from the same wavefront and therefore maintain a constant phase relationship. Furthermore, they emit identical wavelengths since the waves arise from the same source. In this way the **conditions for sustained interference are met**.

The light from the two slits produces a visible pattern on a target screen. The pattern consists of a series of alternating bright and dark parallel bands called **fringes** surrounding a fairly broad central light band.

Young accounted for the interference pattern generally as follows: When the light from slits S_1 and S_2 arrives at a point on the target screen such that **constructive interference** occurs at that location, a **bright line** appears. When light from the two slits **interferes destructively** at any location on the screen, a **dark line** results.

Two waves (that necessarily leave the slits in phase) strike the screen at the **central spot**. Since they travel an equal distance, they arrive in phase and as a result complete constructive interference occurs here and a bright spot is observed.

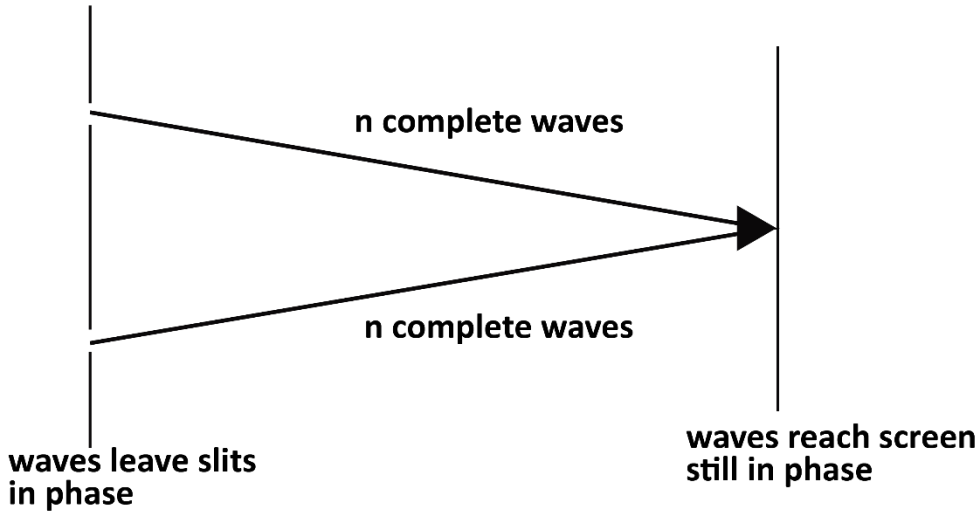

Figure 7.3: *Constructive interference in Young's experiment*

Figure 7.4 illustrates how the **first dark band** away from the central spot is formed. The waves leaving the two slits once again start out being in phase but travel different distances. As a result, one wave is half a wavelength behind the other so that the crest of one wave overlaps with the trough of the other. Complete destructive interference results and a dark fringe is observed.

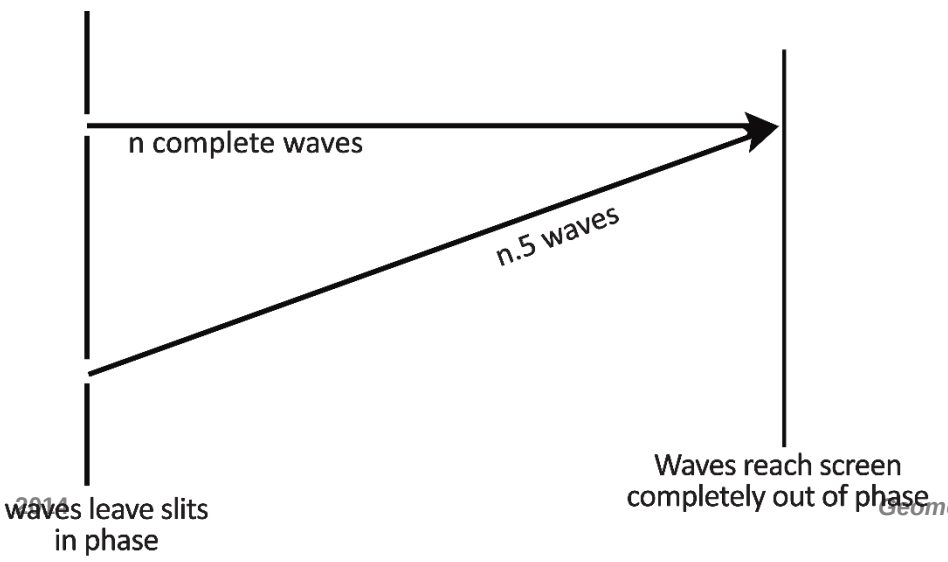

Figure 7.4: *Destructive interference in Young's experiment*

In the following diagram, the waves again start out in phase and travel different distances, but the difference in path length is exactly one wavelength and the waves thus reach the screen in phase. Complete constructive interference results and another (higher order) **bright band** is observed.

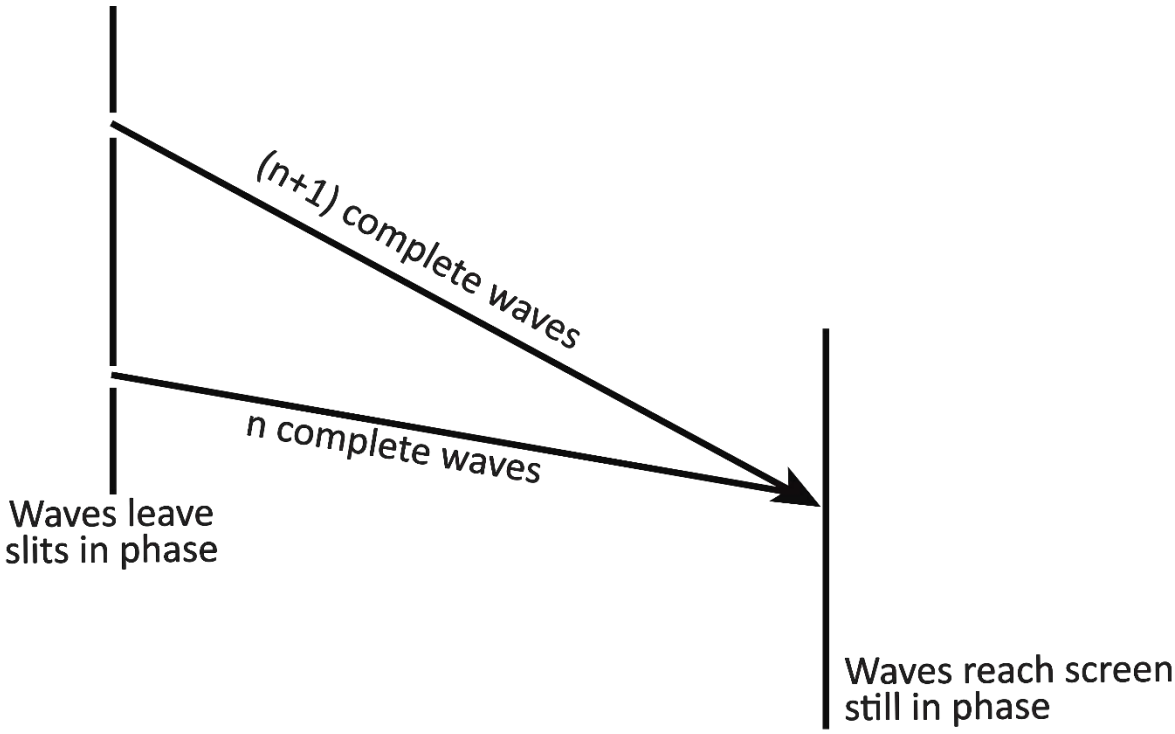

Figure 7.4: *Constructive interference of higher order in Young's experiment*

The foregoing can be summarized as follows:

If the path difference is either zero or some integral multiple of the wavelength, the two waves are in phase at the screen and a bright fringe results. When the path difference is an odd multiple of $\lambda/2$, the two waves arriving at the screen will be 180° out of phase and a dark fringe will result.

What is the effect of wavelength on the interference pattern?

The shorter the wavelength of the incident light, the closer the fringes are to each other, and the narrower they are. Therefore, when the incident light is blue, the fringes are closer together and the light fringes will appear blue. With red incident light the fringes will be further apart and the light bands will appear red.

Bright Fringes

We have previously seen that for bright fringes to be obtained, the difference in path length has to be **nλ**. For the central bright band (first order), n = 0. For the next bright fringe (above or below), n=1 etc.

It can be shown that the difference in path length is $\mathbf{n\lambda = d\sin\theta}$, where θ is the angle between the central bright fringe and the nth bright fringe and **d** is the distance between the slits. It can be shown that this equation is equivalent to:

$$\mathbf{n\lambda = dx/L}$$

x = distance between the bright fringes
L = distance between the double slits and the viewing screen.

Dark Fringes

In this case, the equation becomes:

$$(n + ½)\ \lambda = d\sin\theta = dx/L$$

INTERFERENCE IN THIN FILMS

Interference of light gives rise to many everyday phenomena such as the bright colours reflected from soap bubbles and from thin oil films on water. In these and other cases, the bright colours are a result of interferences between light reflected from the two surfaces of the thin film.
In order to account for these phenomena, we need to consider both factors that influence interference:

(a) **differences in path length** and
(b) **phase reversals as a result of reflection**.

We have already looked at the first factor in the previous section. Now let's consider the second factor.

CHANGE OF PHASE DUE TO REFLECTION

In general:
an electromagnetic wave undergoes a phase change of 180° upon reflection from a medium that has a greater refractive index than the one in which it was traveling.

Let us apply this to a thin film with air on both sides (e.g. the soap film in a soap bubble) as in figure 7.5.

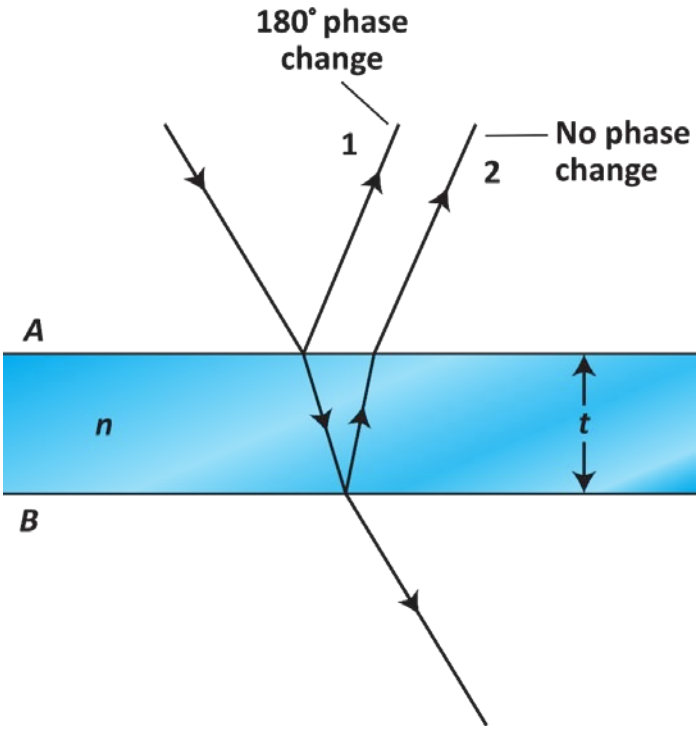

Figure 7.5: *Change of phase due to reflection*

Ray 1, which is reflected from a medium of greater n at the upper surface **A**, (n for the film material is greater than n for air) undergoes a phase change of 180° with respect to the incident wave. On the other hand, ray 2 is reflected from the lower surface **B**. Ray 2 is reflected from a medium of lower refractive index (n for air is less than n for the film material) and therefore this ray undergoes no phase change with respect to the incident wave.

Therefore ray 1 is 180° out of phase with respect to ray 2, which is equivalent to a path difference of $\lambda_n/2$.

NOTE: λ_n is the wavelength of the light in the medium whereas λ is the wavelength in air.

EXAMPLES

B. Soap bubble

This situation is like the one in the Figure 7.5. A thin film of soapy water (n is approximately 1.33) has air on both sides. The water flowing over the surface changes the thickness continually.

The rays from the upper and the lower surfaces interfere with each other. For a bright colour to be seen, the two rays must interfere constructively. To find the requirements for such constructive interference, we need to consider (i) the phase difference due to reflection and (ii) the difference in path length.

We already saw that the two rays are out of phase with respect to each other as a result of reflection. For constructive interference to occur, they need to be completely in phase. Therefore the distance 2t, which is the **difference in path length**, must be equal to a **multiple of $\lambda_n/2$**.

When white light falls on the soap bubble, the path length 2t will equal a multiple of $\lambda_n/2$ for only one wavelength at a given viewing angle. This colour will be seen as very bright. For light viewed at a slightly different angle, the path will be longer or shorter and a different colour will undergo constructive interference. Thus, for an extended (nonpoint) source emitting white light, a series of bright colours will be seen next to one another. Variations in thickness of the soap film will also alter the path length and therefore affect the colour of light that is most strongly reflected. This is how the bright colours in soap films are produced.

B. Oil on Water

We are considering a smooth surface of water on top of which is a thin uniform layer of oil.

Example 1: IF $n_{OIL} > n_{WATER}$

Refer to figure 7.6 to see how the formation of the series of bright colours is explained.

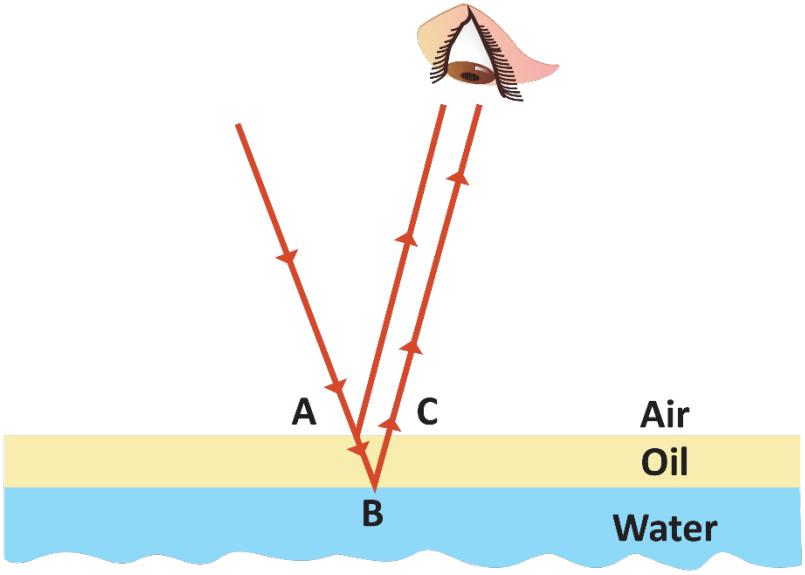

Figure 7.6: *Perception of bright colours when a thin layer of oil is on water*

Part of the incident light (e.g. from the sun or streetlights) is reflected at **A** on the top surface, and part of that transmitted is reflected at **B** on the lower surface. The ray reflected at **A** will undergo a phase reversal and will thus be out of phase with the incident ray. The ray reflected at **B** will not undergo a phase reversal and will thus be in phase with the incident ray. Overall then, the two rays will be out of phase with each other. Note that the ray reflected at the lower surface must travel the extra distance **ABC**.

For a bright colour to be seen the rays must interfere constructively. This can only be possible if the distance **ABC** (the difference in path length) is once again a multiple of $\lambda_n/2$. The two rays will then be completely in phase. For light viewed at a slightly different angle, the path will be longer or shorter and a different colour will undergo constructive interference. Variations in the thickness of the oil will also alter the path length and therefore affect the colour of light that is most strongly reflected. This is why the colours change if the oil is stirred or moved.

Example 2: IF $n_{OIL} < n_{WATER}$

Refer to figure 7,6 again. In this case the ray reflected at **A** will once again be out of phase with the incident ray. However, the ray reflected at **B** will also be out of phase with the incident ray. Thus, only considering phase reversals, the two rays are **in phase** with each other.

The ray reflected at the lower surface must travel the extra distance **ABC**. If the distance **ABC** is equal to one or a whole number of wavelengths, the two waves will interfere constructively and the light will be bright.

When white light falls on such a film, the path **ABC** will equal λ_n (or, $m\lambda_n$ where m is an integer) for only one wavelength at a given viewing angle. This colour will be seen as very bright. For light viewed at a slightly different angle, the path **ABC** will be longer or shorter and a different colour will undergo constructive interference.

Thus, for an extended source emitting white light, a series of bright colours will be seen next to one another. Variations in thickness of the film will also alter the path length **ABC** and therefore affect the colour of light that is most strongly reflected.

ANTI-REFLECTION COATINGS

The bright colours in thin films result from constructive interference. We can use the same idea to cause destructive interference of certain light instead. This would result in certain light not being transmitted. Therefore, if we can design a system where the rays reflected from the two surfaces are completely out of phase, we can prevent unwanted reflections. This is the basis of the design of anti-reflection coatings (ARC).

The reflective loss is given by the **reflectance**. This quantity (either as a fraction or as a percentage) is the ratio of the intensity of reflected light to the intensity of incident light. Therefore, the higher this value is, the greater the amount of light reflected from a surface. Reflectance at the surface of a lens is greater if the index of refraction of the lens substrate is higher.

ANTI-REFLECTIVE COATINGS ON LENSES

Whenever lenses are used, a small percentage of the incident light is reflected from each surface. In compound lens systems, as in telescopes and microscopes, this produces a background of unfocused light, which results in a reduction in the clarity of the final image. There is also a reduction in the intensity of the image, since less light is transmitted through the lenses. Therefore anti-reflection coatings are desirable.

Modern spectacle lenses are often made from materials that have index of refraction values ranging between 1.56 and 1.8. More intense reflections from the lens surfaces can be cosmetically unattractive and may give rise to ghost images that are disturbing to the wearer. Anti-reflection coatings are used to eliminate these reflections.

The amount of reflected light can be considerably reduced by depositing a thin coating on to the surfaces. Some of the light, of average wavelength λ, is then reflected from the air-coating surface. The remainder penetrates the coating and is partially reflected from the coating-glass surface. The amount of light lost to reflection can be reduced if destructive interference occurs between the two reflected beams.

Once again we need to consider both the changes in phase due to reflection and the differences in optical path length. It will be noted that the conditions for destructive interference will depend on the refractive indices for the lens material and the ARC material. This will be illustrated by the examples that follow.

Some of the coatings used are:

MATERIAL	REFRACTIVE INDEX
Cerium oxide	2.20
Zinc sulphide	2.35
Cryolite (Na,Al,F compound)	1.31
Magnesium fluoride	1.38

Table 7.1: *Some commonly used materials for anti-reflective coatings on ophthalmic lenses*

Cryolite and MgF_2 are common low-index films. Since magnesium fluoride is by far the more durable, it is used more frequently. Since the minimum thickness is $t = \lambda/4n$, (**for low refractive index coatings**) the term “quarter wavelength coating” is used.

The wavelength on which the ARC is based, is chosen to be 555 nm since this is the wavelength to which the eye is most sensitive. The thickness is then approximately 100 nm.

A single layer will reduce the reflectance of glass from about 4% to a bit more than 1%. It is **not zero, because the intensity of the light is not the same from the two surfaces.** For that to happen, ***the refractive index of the coating must be equal to the square root of n for the lens.***

The film thickness is increasingly in error as the wavelength is further from 555 nm. Thus, more and more light is reflected approaching the red and blue ends of the spectrum. This explains the characteristic red-blue (purple) appearance of the reflection.

The decrease in reflection over a range of wavelengths for a monolayer ARC can be seen to depend on the angle of incidence. For normally incident light, figure 7.7 shows how the decrease in reflectance is greatest at the design wavelength, 550 nm, and decreases the further we move away from this wavelength.

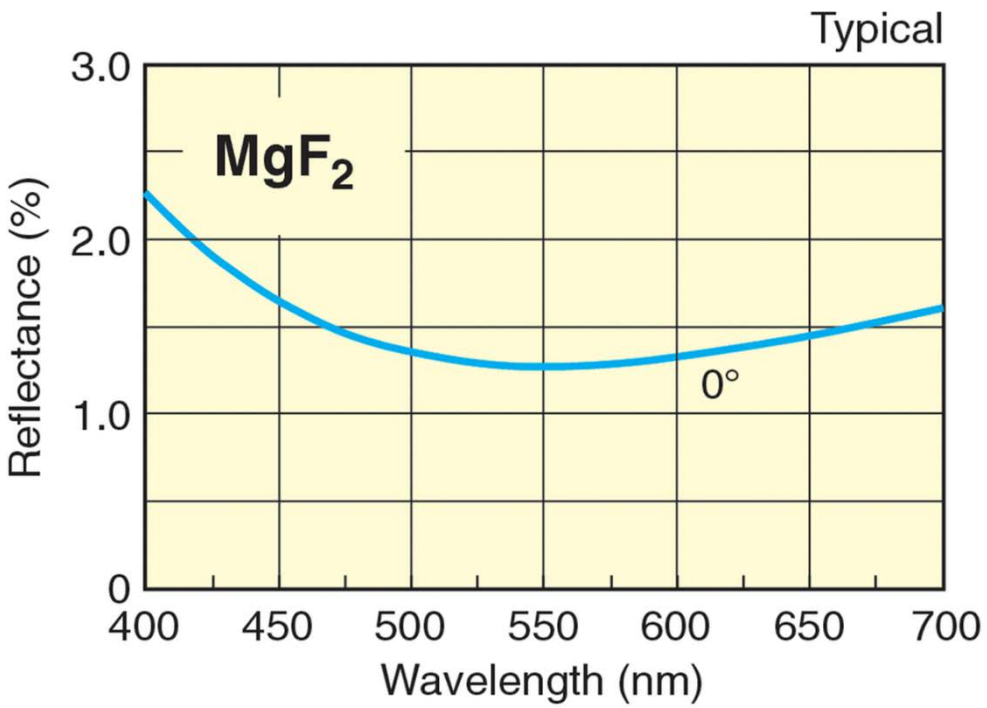

Figure 7.7: *Decrease in reflectance for incident light as a function of wavelength for a monolayer ARC (magnesium fluoride)*

Note that since it is not possible to destroy energy, the light which is not reflected is now transmitted. Hence, such coatings on camera lenses cause a marked increase in image brightness.

A much wider coverage is possible with multiple coatings, called *multilayers*. In a two-layer AR coating, for example, each layer is made ¼λ thick. This is called a quarter-quarter coating. In many three-layer coatings the centre layer is made ½λ thick, the other two layers ¼λ each. Such coatings are widely used; they are effective over most of the visible spectrum. The first, outside layer is often a ¼λ coating of MgF_2, the next is λ/2 zirconium dioxide (index 2.10), and the layer next to the substrate is λ/4 cerium trifluoride (CeF_3, index 1.65) or aluminium oxide (Al_2O_3, index 1.76). Some premium spectacle lenses are manufactured with AR coatings with up to 7 individual layers.